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Diffraction by a highly contrast transparent wedge

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Abstract. The problem of a plane wave diffraction by a highly contrast transparent wedge of an arbitrary opening is studied. With the aid of the Sommerfeld integrals it is reduced to a system of coupled Maliuzhinets' equations. By use of the theory of S-integrals the system of functional equations is transformed to a system of linear equations in a Banach space. In the case of the high contrast of a material inside the wedge in comparison with that in the wedge's exterior the linear equations are solved by means of perturbation theory. Convergence of the corresponding Neumann series is proved. Singularities of the integrands in the Sommerfeld integrals are investigated. Application of the steepest descent method leads to the determination of the reflected, transmitted and diffracted waves. Expressions for the diffraction coefficients are also represented.

1. Introduction

The problem of diffraction by a transparent wedge has attracted the attention of researchers for a long time. Contrary to the problem of diffraction by an impedance wedge solved by Maliuzhinets (1958) in an explicit form, up to now there has been no closed-form solution which is acceptable for applications. The results obtained by Latz (1973) and Kurilko (1968) look very complex and can hardly be used in practice. The solution developed by Kraut and Lehman (1969) is based on progress in the Wiener–Hopf function-theoretic technique. However, the authors could not exploit their results for any applications. Berntsen (1983) considered the problem from the general point of view and developed some new accurate results. He also studied correction to the physical optics approximation. To our mind, however, the numerical implementation for his results as well as a physical analysis are not simple enough for applications. Rawlins (1977) developed the approximate solution which is valid for the refraction index close to unity. He could also obtain the expressions for the far field and used them for the numerical simulation. Interesting results for nearly transparent or thin wedges were represented by Kaminetsky and Keller (1975). They determined simple formulae for the diffraction coefficients which are of great importance in the geometrical theory of diffraction.

Recent progress in the problem under consideration is connected with the works of Budaev (1992, 1995). The author developed the original approach based on Sommerfeld integrals, Maliuzhinets' functional equations and their reduction to singular integral equations. Budaev and Bogy could also obtain numerical results in the corresponding problem of diffraction in an elastic wedge (1996) and in a double wedge (1998). Some promising results have been

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claimed by Osipov†. He applied Kontorovich–Lebedev's transform and reduced the problem in question to integral equations of the Fredholm type.

Among numerous papers devoted to an approximate solution which is based on some physical conceptions (Westwood 1989, 1990, Li *et al* 1992, Booysen and Pistorius 1992) such as physical optics, the paper of Kim *et al* (1991) ought to be mentioned (see also Kim 1997). The corresponding physical motivation is supplemented by a numerical study of the problem. Simple expressions for the diffraction coefficients are also represented. This paper is of particular interest for us since, in principle, it contains the results dealing with a highly contrast transparent wedge. Diffraction by a highly contrast transparent wedge is the main subject of our treatment.

We introduce the parameter λ which is called the parameter of contrast. In the case of two different acoustic media outside and inside a wedge this parameter is the ratio of the densities ρ_1 and ρ_2

$$\lambda = \frac{\rho_1}{\rho_2}.$$

If one studies the electromagnetic problem, the equality $\lambda = \epsilon_1/\epsilon_2$ defines this parameter for the H-polarization and $\lambda = \mu_1/\mu_2$ is for the E-polarization, where ϵ_i and μ_i, $i = 1, 2$ are the permittivities and permeabilities of the media respectively. We say that a wedge is highly contrast if the parameter of contrast is small, i.e.

$$\lambda \ll 1.$$

Another important parameter of the problem is the ratio of the wavenumbers k_1 and k_2 outside and inside the wedge respectively, $\gamma = k_1/k_2$. In this paper our approach enables us to consider λ and γ as independent parameters. In particular, the results which follow from perturbation theory are valid for a sufficiently small λ but for arbitrary values of γ, $0 < \gamma < 1$. On the other hand, the results are also valid if the parameters λ and γ correlate as, for example, in the corresponding electromagnetic problem.

It is worth noticing that the perturbation theory which is used in this paper has been applied to solving coupled Maliuzhinets equations in the works of Lyalinov (1994), of Pelosi *et al* (1998) and in some others.

In section 2 we formulate the problem. By use of the Sommerfeld integrals we reduce it to a system of Maliuzhinets' functional equations for two unknown functions which are in the integrands of the Sommerfeld integrals representing a solution inside and outside the wedge.

In section 3, with the aid of the S-integrals we reduce the functional equations to the system of linear equations in a Banach space. The linear equations contain a small parameter of contrast λ and can be solved by the perturbation method. We also discuss the limiting problems as $\lambda = 0$.

In section 4 we study the singularities of the integrands in the Sommerfeld integrals. Application of the saddle-point technique leads to the nonuniform expressions for the reflected, transmitted and diffracted waves in the far field. The formulae for the diffraction coefficients are also derived. We consider them as one of the main results of this paper. A uniform asymptotic formula for the wave field in the exterior of the wedge is briefly discussed.

Appendices are devoted to the technical results such as an extention of the theory of S-integrals, proof of the boundedness of the operators in the linear equations and formulae for analytic continuations of the spectral functions.

References do not form a complete list; however, they reflect our preferences and knowledge of up-to-date publications on the subject.

† These results can be found in the work: A V Osipov, Thesis for the Doctor of Science Degree, 1995, St Petersburg University.

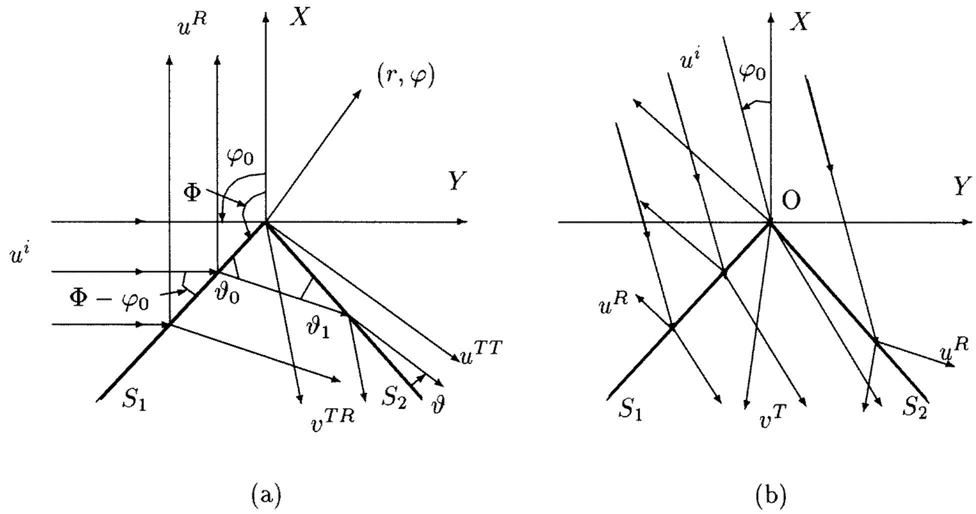


Figure 1. Diffraction by a transparent wedge (*a*) $\pi - \Phi < \varphi_0 < \Phi$, (*b*) $0 < \varphi_0 < \pi - \Phi$.

2. Formulation of the problem and reduction to the system of Maliuzhinets' functional equations

2.1. Formulation

The wave fields u and v satisfy the Helmholtz equations

$$(\triangle + k_1^2)u = 0 \qquad \triangle = \partial_x^2 + \partial_y^2 \tag{1}$$

in the angle (figure 1)

$$\Gamma(-\Phi, \Phi) = \{(r, \varphi) : r > 0, -\Phi < \varphi < \Phi\} \qquad \pi/2 < \Phi < \pi$$

and

$$(\triangle + k_2^2)v = 0 \tag{2}$$

in the supplementary angle

$$\Gamma(\Phi, 2\pi - \Phi) = \{(r, \varphi) : r > 0, \Phi < \varphi < 2\pi - \Phi\}$$

respectively, where (r, φ) are the polar coordinates. The time dependence $\exp(-\mathrm{i}\omega t)$ is used and suppressed in the following. The wave fields are excited by the incident plane wave

$$u^{\mathrm{i}}(k_1 r, \varphi) = \exp(-\mathrm{i}k_1 r \cos(\varphi - \varphi_0)) \tag{3}$$

and satisfy the boundary conditions

$$u|_{S_{1,2}} = v|_{S_{1,2}} \tag{4}$$

$$\partial_\varphi u|_{S_{1,2}} = \lambda \partial_\varphi v|_{S_{1,2}} \tag{5}$$

where S_1 and S_2 are the wedge's faces, λ is the parameter of contrast. The wave field should also satisfy the radiation condition at infinity. This condition implies that, if from u and v we substract the geometrical optics waves u^g and v^g given rise due to reflections and transmissions (including multiple ones) in $\Gamma(-\Phi, \Phi)$ and in $\Gamma(\Phi, 2\pi - \Phi)$ respectively, the remainders should behave themselves as outgoing cylindrical wave as $k_{1,2}r \to \infty$ (see Petrashen' and Budaev 1986)

$$u - u^g = \frac{C_1(\varphi)}{\sqrt{k_1 r}} e^{ik_1 r}\left(1 + O\left(\frac{1}{k_1 r}\right)\right)$$
$$v - v^g = \frac{C_2(\varphi)}{\sqrt{k_2 r}} e^{ik_2 r}\left(1 + O\left(\frac{1}{k_2 r}\right)\right). \tag{6}$$

Instead of radiation conditions (6) one can exploit the principle of limiting absorption (see Rawlins 1977, Berntsen 1986). We also impose the Meixner's condition at the edge

$$u = \text{const} + O(r^\delta) \qquad v = \text{const} + O(r^\delta) \qquad \delta > 0 \tag{7}$$

where δ is the smallest positive root of the equation (Budaev 1992)

$$\cos(\delta\Phi)\sin(\delta\bar{\Phi}) - \lambda\sin(\delta\Phi)\cos(\delta\bar{\Phi}) = 0 \qquad \text{or} \qquad \cot(\delta\Phi) - \lambda\cot(\delta\bar{\Phi}) = 0 \tag{8}$$

where $\bar{\Phi} = \pi - \Phi$. For a small λ, the desired solution of equations (8) is given by the expression

$$\delta = \frac{\pi}{2\Phi} - \frac{1}{2}\lambda\sin(\pi/\Phi) + O(\lambda^2) \tag{9}$$

which means that

$$\delta < \mu \tag{10}$$

where $\mu = \pi/(2\Phi)$ and $\pi/2 < \Phi < \pi$. It is well known that in the case of the Neumann boundary condition on the hard wedge's faces the singularity of the first derivative of a solution at the tip is $O(r^{\mu-1})$. From (9) and (10) it follows that in the case of a transparent wedge this singularity is stronger, $O(r^{\delta-1})$.

2.2. Sommerfeld integrals and problem for the functional equations

Solutions of the Helmholtz equations (1) and (2) are sought in the form of the Sommerfeld integrals

$$u(k_1 r, \varphi) = \frac{1}{2\pi i}\int_{C_\alpha} f(\alpha + \varphi)\exp(-ik_1 r\cos\alpha)\,d\alpha \tag{11}$$

$$v(k_2 r, \varphi) = \frac{1}{2\pi i}\int_{C_\beta} g(\beta + \varphi)\exp(-ik_2 r\cos\beta)\,d\beta \tag{12}$$

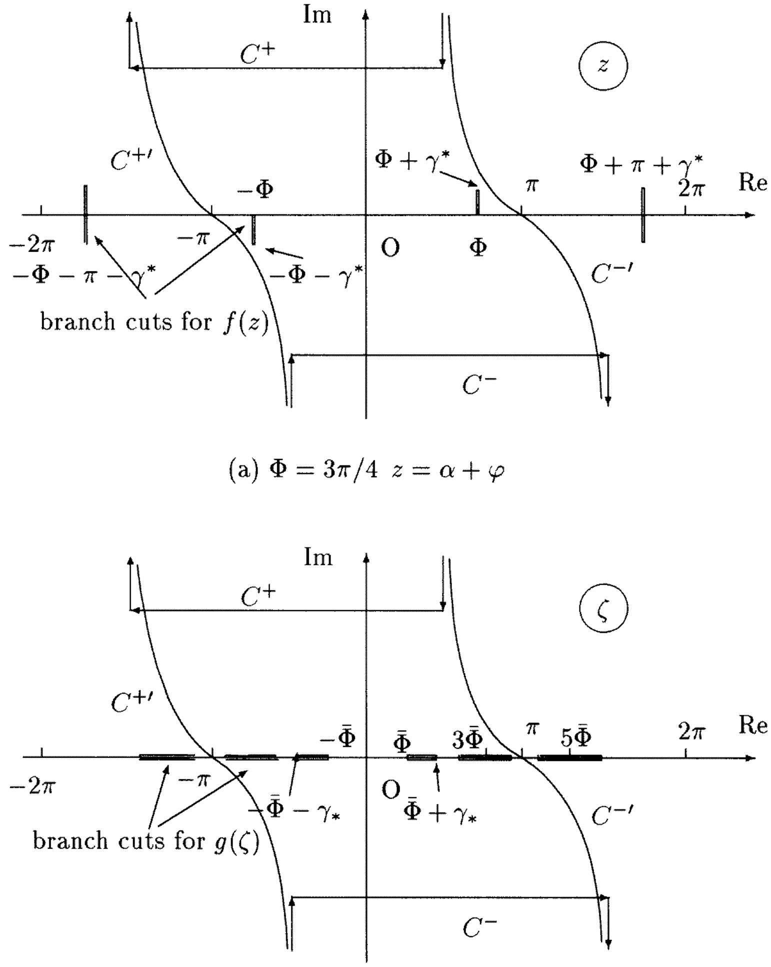
where $C_\alpha = C^+ \cup C^-$ and $C_\beta = C^+ \cup C^-$ are the well known double loop contours (figures 2(*a*) and (*b*)) (see also Maliuzhinets 1958) on α- and β-planes respectively. The spectral functions f and g are to be chosen in order to satisfy boundary conditions (4) and (5). Substituting the integrals (11) and (12) into the boundary conditions, we have

$$\int_{C_\alpha} \exp(-ik_1 r\cos\alpha) f(\alpha - (-1)^j\Phi)\,d\alpha = \int_{C_\beta} \exp(-ik_2 r\cos\beta) g(\beta + (-1)^j\bar{\Phi})\,d\beta \tag{13}$$

$$\int_{C_\alpha} \exp(-ik_1 r\cos\alpha) f'(\alpha - (-1)^j\Phi)\,d\alpha = \int_{C_\beta} \exp(-ik_2 r\cos\beta)\lambda g'(\beta + (-1)^j\bar{\Phi})\,d\beta \tag{14}$$

$j = 1, 2$. We integrate by parts in (14), thus obtaining

$$\int_{C_\alpha} \exp(-ik_1 r\cos\alpha) f(\alpha - (-1)^j\Phi)\gamma\sin\alpha\,d\alpha$$
$$= \int_{C_\beta} \exp(-ik_2 r\cos\beta) g(\beta + (-1)^j\bar{\Phi})\lambda\sin\beta\,d\beta. \tag{15}$$



(a) $\Phi = 3\pi/4 \;\; z = \alpha + \varphi$

(b) $\bar{\Phi} = \pi/4 \;\; \zeta = \beta - \bar{\varphi}$

Figure 2. Sommerfeld contours and branch cuts of the spectral functions.

In order to derive functional equations with the aid of the Maliuzhinets' theorem it is necessary to have the same exponential factors in the integrands of (13) and (15). For this we introduce the new variable of integration α in the right-hand sides of equalities (13) and (15) in accordance with the equation (see also Petrashen' and Budaev 1986)

$$\cos\beta = \gamma\cos\alpha \tag{16}$$

which defines β as a function of α, $\beta = b(\alpha) = \arccos(\gamma\cos\alpha)$ on the α-plane with appropriate branch cuts (figure 3(*a*)). The properties of the map $b(\alpha)$ and its inverse $a(\beta) = \arccos(\gamma^{-1}\cos\beta)$ are very important for further analysis so we discuss them in some details. The functions $b(\alpha)$ and $a(\beta)$ are regular on the complex planes C^* and C_* respectively with the branch cuts shown in figures 3(*a*) and (*b*), where $\gamma^* = \mathrm{i}\,\mathrm{arccosh}(\gamma^{-1})$ and $\gamma_* = \arccos(\gamma)$. The branches are fixed by the condition: the part $[\gamma^* + \mathrm{i}0, \mathrm{i}\infty)$ of the imaginary axis in C^* is univalently mapped onto the part $[+\mathrm{i}0, \mathrm{i}\infty)$ of the imaginary axis in C_*.

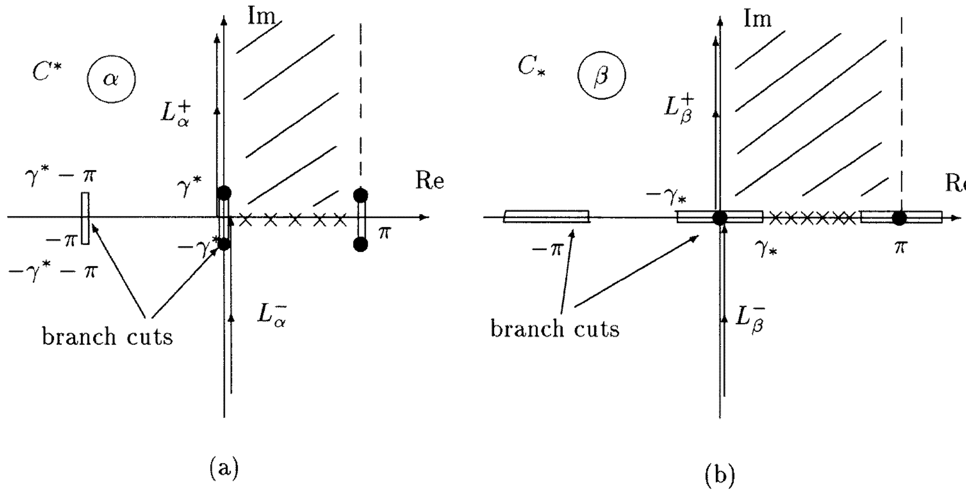


Figure 3. The complex planes C^* and C_*.

This condition preserves the natural property $\lim_{\gamma\to 1-0} b(a) = a$. The conformal function $b(\alpha)$ maps the shaded semistrip in figure 3(a) onto the shaded one in figure 3(b). The following properties of $b(\alpha)$ are easily verified:

$$\begin{aligned} & b(\alpha) = -b(-\alpha) \qquad b'(\alpha) = b'(-\alpha) \\ & b(\bar{\alpha}) = \bar{b}(\alpha) \\ & b(\alpha + \pi n) = b(\alpha) + \pi n \qquad n \in \mathbb{Z} \\ & b(\alpha) = \alpha + \mathrm{i}\log\gamma + \mathrm{O}(1/\cos\alpha) \qquad \alpha \to +\mathrm{i}\infty \\ & b(\pi/2 + \mathrm{i}t_0) = \pi/2 + \mathrm{i}s(t_0) \qquad t_0 \in \mathbb{R} \quad s \in \mathbb{R}. \end{aligned} \tag{17}$$

In the last equality $s(t_0)$ is a real function of its argument. Properties similar to (17) are valid for $a(\beta)$. From equation (16) we obtain

$$\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = \gamma\frac{\sin\alpha}{\sin\beta} = \frac{\gamma\sin\alpha}{\sqrt{1-\gamma^2\cos^2\alpha}}. \tag{18}$$

Performing the mentioned change of variables in the right-hand sides of (13) and (15), then, exploiting (18) and the well known results on inversion of Sommerfeld integrals, we come to the coupled system of functional equations

$$\begin{aligned} & f(\alpha - (-1)^j\Phi) - f(-\alpha - (-1)^j\Phi) = \frac{\gamma\sin\alpha}{\sqrt{1-\gamma^2\cos^2\alpha}} \\ & \qquad \times[g(b(\alpha) + (-1)^j\bar{\Phi}) - g(-b(\alpha) + (-1)^j\bar{\Phi})] \\ & f(\alpha - (-1)^j\Phi) + f(-\alpha - (-1)^j\Phi) = \lambda[g(b(\alpha) + (-1)^j\bar{\Phi}) + g(-b(\alpha) + (-1)^j\bar{\Phi})] \end{aligned} \tag{19}$$

where $j = 1, 2$. The complex variable α belongs to the complex plane C^*. Budaev (1992) could reduce a system analogous to (19) to a singular integral equation and studied it. We shall follow a different way which seems more convenient to us and enables us to develop relatively simple expressions for the diffraction coefficients when λ is sufficiently small.

We multiply the first equation in (19) by $\lambda\sqrt{1-\gamma^2\cos^2\alpha}$ and the second one by $\gamma\sin\alpha$ and sum them up. Then, multiplying the first equation by $\sqrt{1-\gamma^2\cos^2\alpha}$ and the second one by the same factor, we also sum them up. As a result, we come to the equivalent system of functional equations

$$\left(\gamma\sin\alpha+\lambda\sqrt{1-\gamma^2\cos^2\alpha}\right)f(\alpha-(-1)^j\Phi)+\left(\gamma\sin\alpha-\lambda\sqrt{1-\gamma^2\cos^2\alpha}\right)$$
$$\times f(-\alpha-(-1)^j\Phi)=2\gamma\lambda\sin\alpha\, g(b(\alpha)+(-1)^j\bar{\Phi})$$
$$\left(\gamma\sin\alpha+\lambda\sqrt{1-\gamma^2\cos^2\alpha}\right)g(b(\alpha)+(-1)^j\bar{\Phi})+\left(\gamma\sin\alpha-\lambda\sqrt{1-\gamma^2\cos^2\alpha}\right)$$
$$\times g(-b(\alpha)+(-1)^j\bar{\Phi})=2\sqrt{1-\gamma^2\cos^2(\alpha)}\,f(\alpha-(-1)^j\Phi)$$

with $j=1,2$. Elementary transformations enable us to obtain

$$\begin{aligned} f(\alpha-(-1)^j\Phi) &= -R(\alpha,\gamma,\lambda)f(-\alpha-(-1)^j\Phi)+\lambda T(\alpha,\gamma,\lambda)g(b(\alpha)+(-1)^j\bar{\Phi})\\ g(\beta+(-1)^j\bar{\Phi}) &= -R(\beta,\gamma^{-1},\lambda^{-1})g(-\beta+(-1)^j\bar{\Phi})\\ &\quad+\lambda^{-1}T(\beta,\gamma^{-1},\lambda^{-1})f(a(\beta)-(-1)^j\Phi)\end{aligned} \tag{20}$$

where $j=1,2$, $\alpha\in C^*$, $\beta\in C_*$; R and $T=1+R$ are the so-called reflection and transmission coefficients

$$R(\alpha,\gamma,\lambda)=\frac{\gamma\sin\alpha-\lambda\sqrt{1-\gamma^2\cos^2\alpha}}{\gamma\sin\alpha+\lambda\sqrt{1-\gamma^2\cos^2\alpha}} \tag{21}$$

$$T(\alpha,\gamma,\lambda)=\frac{2\gamma\sin\alpha}{\gamma\sin\alpha+\lambda\sqrt{1-\gamma^2\cos^2\alpha}}. \tag{22}$$

It is also convenient to write system (19) as follows:

$$\begin{aligned} & f(\alpha-(-1)^j\Phi)+f(-\alpha-(-1)^j\Phi)\\ &\quad=\lambda(g(b(\alpha)+(-1)^j\bar{\Phi})+g(-b(\alpha)+(-1)^j\bar{\Phi}))\\ & g(\beta+(-1)^j\bar{\Phi})-g(-\beta+(-1)^j\bar{\Phi})\\ &\quad=\frac{\sin\beta}{\sqrt{\gamma^2-\cos^2\beta}}(f(a(\beta)-(-1)^j\Phi)-f(-a(\beta)-(-1)^j\Phi)\end{aligned} \tag{23}$$

with $j=1,2$. Both equivalent forms (20) and (23) will be used for analysis.

The system of functional equations (23) should be supplemented by additional conditions which follow from radiation condition (6) and from Meixner's conditions (7) at the edge; see, for example, Budaev (1992), Petrashen' and Budaev (1986). Since the behaviour of a Sommerfeld integral as $r\to 0$ is determined by the asymptotics of the spectral function as α (or β) $\to\infty$, from the Meixner's conditions (7) we obtain

$$|f(\alpha)-f(\pm\mathrm{i}\infty)|\leqslant \mathrm{const}\exp(-\delta|\mathrm{Im}\,\alpha|)\qquad \alpha\to\pm\mathrm{i}\infty \tag{24}$$

$$|g(\beta)-g(\pm\mathrm{i}\infty)|\leqslant \mathrm{const}\exp(-\delta|\mathrm{Im}\,\beta|)\qquad \beta\to\pm\mathrm{i}\infty. \tag{25}$$

The spectral function $g(\beta)$ must be regular in the basic strip $\Pi_\beta(-\bar{\Phi},\bar{\Phi})=\{\beta\in C_*:-\bar{\Phi}<\mathrm{Re}\,\beta<\bar{\Phi}\}$. The function $f(\alpha)$ must be regular in the strip $\Pi_\alpha(-\Phi,\Phi)=\{\alpha\in C^*:-\Phi<\mathrm{Re}\,\alpha<\Phi\}$ with the exception of one pole at $\varphi=\varphi_0$, $(0<\varphi_0<\Phi)$. The residue at this pole is determined by the amplitude of the incident wave (3) so that $\mathrm{res}_{\varphi_0}f(\alpha)=1$. It is known that the radiation conditions imply the above-mentioned properties of $f(\alpha)$ and $g(\beta)$, namely

$$f(\alpha)-1/(\alpha-\varphi_0)\qquad \text{is regular in}\quad \Pi_\alpha(-\Phi,\Phi) \tag{26}$$

$$g(\beta)\qquad \text{is regular in}\quad \Pi_\beta(-\bar{\Phi},\bar{\Phi}). \tag{27}$$

Because it is well known that we can add a constant to the integrand of the Sommerfeld integral without changing its value, so it is convenient and always possible to choose

$$f(\mathrm{i}\infty)=-f(-\mathrm{i}\infty) \tag{28}$$

$$g(\mathrm{i}\infty)=-g(-\mathrm{i}\infty). \tag{29}$$

Conditions (28) and (29) enable us to exclude some freedom in determination of the spectral functions: continuity of the wave field at the edge and the second equation in (23) imply the equality

$$g(\mathrm{i}\infty) = f(\mathrm{i}\infty). \tag{30}$$

It is useful to notice that values and all singularities of the spectral functions outside the basic strips can be calculated from functional equations (20) or (23) provided that we have already found their solution in the basic strips.

Although some results on explicit solvability of coupled Maliuzhinets' equations are known (Lyalinov 1997, Bernard 1998), the problem for functional equations (23) can hardly be solved in an exact form. However, we intend to reduce the problem (23)–(29) to a system of linear equations in a Banach space and, then, to solve it by a perturbation method for a small λ. In order to obtain the mentioned system of linear equations we should have an opportunity to invert some difference operators attributed to the system (23). Such an inversion can be performed by use of some generalization of the theory of S-integrals proposed by Tuzhilin (1973). The corresponding results are represented in appendix A.

3. Reduction to a system of linear equations

First, instead of $f(\alpha)$ and $g(\beta)$ we introduce new unknown functions $F(\alpha)$ and $G(\beta)$ defined by the equalities

$$f(\alpha) = \sigma_{\varphi_0}(\alpha)F(\alpha) \qquad \sigma_{\varphi_0}(\alpha) = \frac{\mu\cos\mu\alpha}{\sin\mu\alpha - \sin\mu\varphi_0} \tag{31}$$

$$g(\beta) = G(\beta). \tag{32}$$

Note that $\sigma_{\varphi_0}(\alpha)$ defined in (31) has the required pole at $\alpha = \varphi_0$, then, taking into account $\mathrm{res}_{\varphi_0} f(\alpha) = 1$, we have

$$F(\varphi_0) = 1. \tag{33}$$

The function $\sigma_{\varphi_0}(\alpha)$ represents the spectral function in the diffraction problem by a wedge with hard faces, i.e., with Neumann boundary conditions on them. This problem is the limiting case for a highly contrast transparent wedge as $\lambda \to 0$: $F(\alpha) \to 1$. With the aid of formulae (31) and (32) we write system (23) as follows:

$$F(\alpha - (-1)^j\Phi) - F(-\alpha - (-1)^j\Phi) = (-\lambda)\frac{\cos\mu\alpha + (-1)^j\sin\mu\varphi_0}{\mu\sin\mu\alpha} \times [G(b(\alpha) + (-1)^j\bar{\Phi}) + G(-b(\alpha) + (-1)^j\bar{\Phi})] \tag{34}$$

$$G(\beta + (-1)^j\bar{\Phi}) - G(-\beta + (-1)^j\bar{\Phi}) = \frac{\sin\beta}{\sqrt{\gamma^2 - \cos^2\beta}}\frac{(-\mu)\sin\mu a(\beta)}{\cos\mu a(\beta) + (-1)^j\sin\mu\varphi_0} \times [F(a(\beta) - (-1)^j\Phi) + F(-a(\beta) - (-1)^j\Phi)] \tag{35}$$

with $j = 1, 2$. Note that $F(\alpha)$ satisfies condition (24). From (26) we have that $F(\alpha)$ is regular in $\Pi_\alpha(\Phi, \Phi)$. Condition (28) is changed by

$$F(\mathrm{i}\infty) = F(-\mathrm{i}\infty). \tag{36}$$

In accordance with (32) the function $G(\beta)$ meets the same conditions as $g(\beta)$. The right-hand side of equations (34) tends to zero exponentially as $\alpha \to \mathrm{i}\infty$ whereas the right-hand of (35) is a constant at infinity. We apply the theory of S- and s-integrals discussed in appendix A and invert the difference operators in the left-hand sides of (34) and (35) respectively, thus obtaining

$$F(\alpha) = A_F + \frac{\mathrm{i}\lambda}{8\Phi} \sum_{j=1}^{2} \int_{L_\alpha} (-1)^j \mu^{-1} \frac{\cos\mu\tau + (-1)^j \sin\mu\varphi_0}{\cos\mu\tau + (-1)^j \sin\mu\alpha} \times [G(b(\tau) + (-1)^j \bar{\Phi}) + G(-b(\tau) + (-1)^j \bar{\Phi})]\,\mathrm{d}\tau \tag{37}$$

where $\alpha \in \Pi_\alpha(-\Phi, \Phi)$, and

$$G(\beta) = A_G - \frac{\mathrm{i}\sin\bar{\mu}\beta}{8\bar{\Phi}} \sum_{j=1}^{2} \int_{L_\beta} \frac{D_j(\tau, F)}{\cos\bar{\mu}\tau + (-1)^j \sin\bar{\mu}\beta}\,\mathrm{d}\tau \qquad \beta \in \Pi_\beta(-\bar{\Phi}, \bar{\Phi}) \tag{38}$$

where

$$D_j(\tau, F) = \frac{\mu\sin\tau}{\sqrt{\gamma^2 - \cos^2\tau}} \frac{\sin\mu a(\tau)\tan\bar{\mu}\tau}{\cos\mu a(\tau) + (-1)^j \sin\mu\varphi_0} \times [F(a(\tau) - (-1)^j \Phi) + F(-a(\tau) - (-1)^j \Phi)]. \tag{39}$$

The arbitrary constants A_F and A_G which are solutions of the corresponding homogeneous difference equations can be explicitly calculated. We put $\beta \to \mathrm{i}\infty$ in both sides of (38) and take into account (39). After some computations we find

$$A_G = \frac{\mathrm{i}}{8\bar{\Phi}} \int_{L_\beta} (D_2(\tau, F) - D_1(\tau, F))\,\mathrm{d}\tau.$$

The constant A_F is easily determined from (33). As a result, from (37) and (38) we obtain

$$F(\alpha) = 1 + \lambda(K_1 G)(\alpha) \tag{40}$$

$$G(\beta) = (K_2 F)(\beta) \tag{41}$$

where

$$(K_1\xi)(\alpha) = \frac{\mathrm{i}}{8\Phi} \sum_{j=1}^{2} \int_{L_\alpha} \frac{(-1)^j}{\mu} \left\{ \frac{\cos\mu\tau + (-1)^j \sin\mu\varphi_0}{\cos\mu\tau + (-1)^j \sin\mu\alpha} - 1 \right\} \times [\xi(b(\tau) + (-1)^j \bar{\Phi}) + \xi(-b(\tau) + (-1)^j \bar{\Phi})]\,\mathrm{d}\tau \tag{42}$$

for $\alpha \in \Pi_\alpha(\Phi, \Phi)$. Otherwise, $(K_1\xi)(\alpha)$ is defined by the corresponding analytic continuation of S-integrals; see (68), (71) and (72), and

$$(K_2\eta)(\beta) = \frac{\mathrm{i}}{8\bar{\Phi}} \sum_{j=1}^{2} \int_{L_\beta} \frac{(-1)\mu\sin\tau}{\sqrt{\gamma^2 - \cos^2\tau}} \frac{\sin\mu a(\tau)\tan\bar{\mu}\tau}{\cos\mu a(\tau) + (-1)^j \sin\mu\varphi_0} [\eta(a(\tau) - (-1)^j \Phi) + \eta(-a(\tau) - (-1)^j \Phi)] \left(\frac{\sin\bar{\mu}\beta}{\cos\bar{\mu}\tau + (-1)^j \sin\bar{\mu}\beta} - (-1)^j \right) \mathrm{d}\tau \tag{43}$$

for $\beta \in \Pi_\beta(\bar{\Phi}, \bar{\Phi})$. Otherwise, $(K_2\eta)(\beta)$ is defined by the corresponding analytic continuation of s-integrals; see (69).

3.1. Spaces of solutions.

Let us consider the linear spaces A_1 and A_2 which consist of functions regular in the strips (figure 4) Π_1 and Π_2, respectively; $\varepsilon > 0$ is small. We also assume that functions $\xi \in A_1$ and $\eta \in A_2$ can be extended as continuous functions on the boundaries of the strips Π_1 and Π_2, respectively. (The sides of the branch cuts shown in figures 4(*a*) and (*b*) belong to the

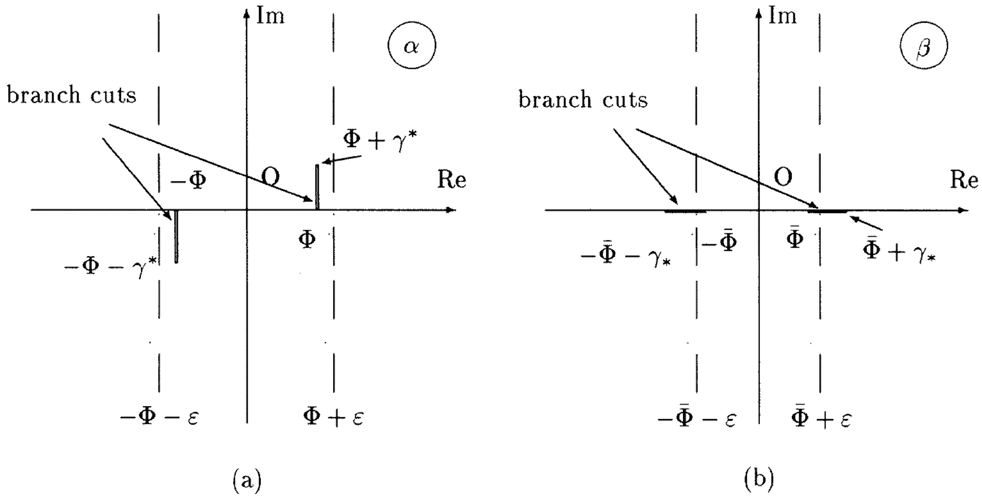


Figure 4. The strips (*a*) Π_1, (*b*) Π_2.

boundaries.) We force these functions satisfy the estimates (24) and (25) as well as conditions (36) and (29) correspondingly. We endow the linear space A_1 with a weighted norm

$$\|\xi; A_1\| = \sup_{\alpha\in\Pi_1}|\xi(\alpha)| + \sup_{\alpha\in\Pi_1}|(\xi(\alpha) - \xi(\mathrm{i}\infty))\nu_\delta(\alpha)| \tag{44}$$

where $\nu_\delta(\alpha)$ is a weight function: it has neither zeros nor poles in Π_1, is regular there and satisfies the estimate

$$|\nu_\delta(\alpha)| = \mathrm{O}(\exp(\delta|\mathrm{Im}\,\alpha|)) \qquad \text{as} \quad |\mathrm{Im}\,\alpha| \to \infty \quad \text{in} \quad \Pi_1.$$

The linear space A_1 endowed by the norm (44) is a Banach space. We introduce a norm in A_2

$$\|\eta; A_2\| = \sup_{\beta\in\Pi_2}|\eta(\beta)| + \sup_{\beta\in\Pi_2}|(\eta(\beta) - \kappa(\beta)\eta(\mathrm{i}\infty))\nu_\delta(\beta)| \tag{45}$$

where $\kappa(\beta)$ is a regular in Π_2 function which is continuous up to the boundary and has the asymptotics

$$\kappa(\beta) = \pm 1 + \mathrm{O}(\exp(-\delta_1|\mathrm{Im}\,\beta|)) \qquad \mathrm{Im}\,\beta \to \pm\infty \quad \delta_1 > \delta.$$

The only role of this function is to ensure an appropriate sign in the second term of (45) as $\beta \to \pm\mathrm{i}\infty$. Note that, contrary to $\xi(\alpha)$, $\eta(\beta)$ has opposite signs at $\beta \to \pm\mathrm{i}\infty$ and we are forced to use $\kappa(\beta)$ in order to have $\kappa(\beta)\eta(\mathrm{i}\infty) \to \pm\eta(\mathrm{i}\infty)$ as $\beta \to \pm\mathrm{i}\infty$. It is not difficult to produce such a function in an explicit form. A_2 is a Banach space with the norm (45).

The linear operator K_1 in (40) maps functions from A_2 into A_1 and the operator K_2 in (41) maps functions from A_1 into A_2. Moreover, the following proposition is valid.

Proposition 3.1. *The system of functional equations (34) and (35) for the functions F and G, satisfying (33), (36), (24), and (25), (29) respectively, which are regular in the corresponding strips Π_1 and Π_2, is equivalent to the linear system of equations (40) and (41) with the operators K_1 and K_2 defined above.*

We could demonstrate that linear equations (40) and (41) follow from the corresponding system of functional equations. In order to prove the equivalence it is sufficient to verify that any solution of (40) and (41) is a solution of (34) and (35). This can be done by the direct substitution of (40) and (41) into the functional equations (34) and (35) and by use of S-(s-)integrals; see the lemmas in appendix A.

3.2. Perturbation series and their convergence

For sufficiently small λ the solution of system (40) and (41) can be constructed by means of successive approximations. Consider the following recurrence procedure

$$G_{m-1} = K_2 F_{m-1} \qquad m = 1, 2, 3, \ldots \tag{46}$$

$$F_m = K_1 G_{m-1} \qquad F_0 = 1 \tag{47}$$

then, if the series

$$F = F_0 + \lambda F_1 + \lambda^2 F_2 + \cdots \tag{48}$$

$$G = G_0 + \lambda G_1 + \lambda^2 G_2 + \cdots \tag{49}$$

with F_m and G_m, defined in (46) and (47), converge, they give a solution of system (48) and (49). It is obvious that convergence of series (48) and (49) in the corresponding Banach spaces follows from the inequality

$$\lambda \|K_1\| \|K_2\| < 1. \tag{50}$$

We can always take λ sufficiently small so that the inequality (50) holds provided that K_1 and K_2 are bounded operators. The problem of convergence amounts to the boundedness of the operators K_1 and K_2.

Proposition 3.2. *The operators K_1 and K_2 defined by expressions (42) and (43) respectively are bounded if $\delta < \mu$ (see (10)).*

The proof of the former proposition is tedious and is given in appendix B.

3.3. Solutions in the limiting case as $\lambda = 0$

It is natural to expect that in the limiting case $\lambda = 0$ the solution is simplified. Indeed, we have

$$F(\alpha) = 1 \qquad f(\alpha) = \sigma_{\varphi_0}(\alpha)$$

as $\lambda = 0$, which enables us to obtain

$$u(k_1 r, \varphi) = \frac{1}{2\pi \mathrm{i}} \int_{C_\alpha} \sigma_{\varphi_0}(\alpha + \varphi) \exp(-\mathrm{i} k_1 r \cos\alpha)\, \mathrm{d}\alpha.$$

As a result, we observe that in the limiting case the solution coincides with that in the diffraction problem for a hard wedge, i.e., with the Neumann boundary conditions on the wedge's faces (see (5)) as $\lambda = 0$).

For the limiting spectral function in the interior of the wedge we find from (41), (43):

$$g(\beta) = G(\beta) = (K_2 1)(\beta) = \frac{\mathrm{i}}{8\bar{\Phi}} \sum_{j=1}^{2} \int_{L_\beta} \frac{(-2)\mu \sin\tau}{\sqrt{\gamma^2 - \cos^2\tau}} \frac{\sin \mu a(\tau) \tan \bar{\mu}\tau}{\cos \mu a(\tau) + (-1)^j \sin \mu\varphi_0}$$
$$\times \left(\frac{\sin \bar{\mu}\beta}{\cos \bar{\mu}\tau + (-1)^j \sin \bar{\mu}\beta} - (-1)^j \right) \mathrm{d}\tau.$$

Then the Sommerfeld integral (12) with the limiting spectral function $g(\beta)$ represents the solution of the problem in the interior of the wedge with inhomogeneous Dirichlet boundary conditions (4), where the left-hand side $u(k_1 r, \varphi)|_{S_{1,2}}$ in (4) is known as the solution of the corresponding Neumann problem in the exterior of the wedge. The wave field is excited by 'sources' distributed along the boundaries. Note that solutions in the limiting case can be derived directly from equations (23) as $\lambda = 0$.

Proof of convergence of the perturbation series (48) and demonstration of the limiting case are important from the general point of view. In practice, however, we need the first few terms of the expansions in order to obtain the far-field asymptotics. The far-field asymptotics as well as the diffraction coefficients are of the most interest for applications.

4. Singularities of the spectral functions and the geometrical optics field; the diffraction coefficients

4.1. Singularities of the spectral functions

By use of the saddle point technique the calculations of the far-field asymptotics from the corresponding Sommerfeld integrals is traditional: Maliuzhinets' (1958). One should deform the Sommerfeld contours $C^{\pm}_{\alpha,\beta}$ into the steepest descent paths $C^{\pm\prime}$ going through the points $\pm\pi$ (figure 2), $k_{1,2}r \to \infty$. In the process of this deformation some singularirties of the spectral functions $f(\alpha+\varphi)$ in $\Pi_\alpha(-\pi-\Phi, \pi+\Phi)$ and $g(\beta-\bar\varphi)$ in $\Pi_\beta(-\pi-\bar\Phi, \pi+\bar\Phi)$ can be crossed, ($|\varphi| \leqslant \Phi, |\bar\varphi| \leqslant \bar\Phi$). The contribution of each singularity is interpreted from the point of view of the geometrical theory of diffraction: the poles of the integrands give rise to the incident, reflected and transmitted waves, the contributions from the saddle points lead to the expressions for the cylindrical waves outgoing from the edge, contributions from the branch cuts describe the lateral waves. It is useful to notice that, contrary to the reflected and transmitted waves, diffracted cylindrical waves as well as lateral ones cannot be computed with the aid of the simple consideration based on the well known laws of reflecton and transmission accross a transparent boundary. For simplicity we discuss the wedge's opening 2Φ satisfying the constraints $\pi < 2\Phi < 3\pi/2$, which means absence of the multiple reflections inside the wedge. Moreover, we shall consider the right-angle wedge with $\Phi = 3\pi/4$ implying that the other angles of wedge openings $3\pi/2 < 2\Phi < 2\pi$ can be studied analogously. It should be emphasized that the case with the multiple reflections can be also considered, which leads to some quantitative but not qualitative complications of analysis.

We need to know singularities of $f(z)$ in the strip $\Pi_z(-\pi-\Phi, \pi+\Phi)$, $z = \alpha+\varphi$ and those of $g(\zeta)$ in $\Pi_\zeta(-\pi-\bar\Phi, \pi+\bar\Phi)$, $\zeta = \beta-\bar\varphi$. In the leading approximations the expressions for $f(z)$ and $g(\zeta)$ in the basic strips $\Pi_z(-\Phi, \Phi)$ and $\Pi_\zeta(-\bar\Phi, \bar\Phi)$ are given by (see (31), (32) and (48), (49))

$$f(z) = \sigma_{\varphi_0}(z)F(z) = \frac{\mu\cos\mu z}{\sin\mu z - \sin\mu\varphi_0}[1 + \lambda(K_1G_0)(z) + \mathrm{O}(\lambda^2)] \tag{51}$$

$$g(\zeta) = G(\zeta) = G_0(\zeta) + \mathrm{O}(\lambda) = (K_2F_0)(\zeta) + \mathrm{O}(\lambda) \qquad F_0 = 1 \tag{52}$$

respectively, where the explicit formulae for the operators K_1 and K_2 are defined by (42) and (43). Analytic continuations for $f(z)$ and $g(z)$ follow from the functional equations and are represented in appendix C.

4.2. Contributions from the poles: the geometrical optics field

Let us turn to the contributions from poles. We assume that $\pi - \Phi < \varphi_0 < \Phi$ that is only one side of the wedge is illuminated by the incident wave (figure 1(*a*)). The case of the both sides illuminated is considered in the same manner. The denominator of $\sigma_{\varphi_0}(2\Phi - z)$ in the right-hand side of (91) is equal to zero at $z_0 = 2\Phi - \varphi_0$, ($\alpha_0 = 2\Phi - \varphi - \varphi_0$). Calculating the residue contribution of this pole, we obtain the expression for the wave reflected from the wedge's side S_1

$$u^R(k_1r, \varphi) = R(\Phi - \varphi_0, \gamma, \lambda)\exp\{-\mathrm{i}k_1r\cos[\varphi + \varphi_0 - 2\Phi]\}H(\pi - |\varphi + \varphi_0 - 2\Phi|)$$

where $H(x)$ is the Heaviside unit-step function. The contribution from the pole $z_0 = \varphi_0$ gives rise to the incident wave (3).

Since we did not restrict ourselves by the leading approximation in (91) and (92) of appendix C but preserved the first correction for the leading terms, we can compute the doubly transmitted (refracted by the wedge) wave (figure 1(*a*)) which has $\mathrm{O}(\lambda)$ in the domain

$\Gamma(-\Phi, \Phi)$. This wave originates from the pole of the function $G_0(b(z+\Phi)+\bar{\Phi})$ in (92). In formula (92) $(z+\Phi)+\bar{\Phi} \in \Pi_\beta(b(-2\Phi)+\bar{\Phi}, \bar{\Phi})$ and we use expression (94) for analytic continuation of $g(\zeta) = G_0(\zeta) + \mathrm{O}(\lambda)$, defined in the basic strip, onto $\Pi_\beta(b(-2\Phi), -\bar{\Phi})$. We seek the pole of the last summand in (94) which satisfies the equation

$$\cos \mu a(\zeta_0(z_0)+\bar{\Phi}) - \sin \mu\varphi_0 = 0$$

where $\zeta_0(z_0) = b(z_0+\Phi)+\bar{\Phi}$. We find that the real root which is of interest for us fulfils the equation

$$\zeta_0(z_0)+\bar{\Phi} = -b(\Phi-\varphi_0)$$

or

$$b(z_0+\Phi)+\bar{\Phi} = -\bar{\Phi} - b(\Phi-\varphi_0).$$

The last equation can be represented in the form

$$2\bar{\Phi}+\vartheta_0+\vartheta_1 = \pi \tag{53}$$

where $\vartheta_0 = b(\Phi-\varphi_0)$, $\vartheta_1 = b(\vartheta)$, $\vartheta = \pi+\Phi+z_0$, $z_0 \in \Pi_z(-\pi-\Phi, -\Phi)$. Equation (53) expresses the well known geometrical equality for the sum of angles in a triangle (see figure 1(*a*)). The angle $\vartheta(z_0)$ defining the propagation direction of the doubly transmitted wave is determined in accordance with the geometrical optics laws of refraction $\cos\vartheta_1 = \gamma\cos\vartheta$, $\cos\vartheta_0 = \gamma\cos(\Phi-\varphi_0)$ and by use of equality (53). Computing the residues of $f(z)$ at the corresponding pole, we find the expression for the doubly transmitted wave

$$\begin{aligned} u^{TT}(k_1r,\varphi) = {} & T(\Phi-\varphi_0,\gamma\lambda)T(2\bar{\Phi}+b(\varphi-\varphi_0),\gamma^{-1},\lambda^{-1}) \\ & \times \exp\{-\mathrm{i}k_1r\cos[\varphi+\Phi+a(2\bar{\Phi}+b(\Phi-\varphi_0))]\} \\ & \times H(\pi-\varphi-\Phi-a(2\bar{\Phi}+b(\Phi-\varphi_0))). \end{aligned}$$

The doubly transmitted wave does not propagate outside the wedge in the case of the total internal reflection on the second wedge's side S_2; otherwise, the condition of its existence is satisfied: $\vartheta_1 > \arccos\gamma$ or $\pi - 2\bar{\Phi} - b(\Phi-\varphi_0) > \arccos\gamma$.

In the same manner the geometrical optics wave field inside the wedge can be computed. For example, the transmitted reflected wave (figure 1(*a*)) inside the wedge takes the form

$$\begin{aligned} v^{TR}(k_2r,\bar{\varphi}) = {} & T(\Phi-\varphi_0,\gamma,\lambda)R(2\Phi+b(\Phi-\varphi_0),\gamma^{-1},\lambda^{-1}) \\ & \times \exp\{-\mathrm{i}k_2r\cos[\bar{\varphi}+3\bar{\Phi}+b(\Phi-\varphi_0)]\}H(\pi-[3\bar{\Phi}+\bar{\varphi}+b(\Phi-\varphi_0)]). \end{aligned}$$

Consideration of the case when both sides of the wedge are illuminated by the incident wave does not present any difficulties.

4.3. The diffraction coefficients

Let us now turn to the nonuniform expressions for the diffraction coefficients $D_1(\varphi)$ and $D_2(\bar{\varphi})$ defined as follows

$$u^c(k_1r,\varphi) = D_1(\varphi)\frac{\mathrm{e}^{\mathrm{i}k_1r+\mathrm{i}\pi/4}}{\sqrt{2\pi k_1 r}}(1+\mathrm{O}((k_1r)^{-1})) \tag{54}$$

$$v^c(k_2r,\bar{\varphi}) = D_2(\bar{\varphi})\frac{\mathrm{e}^{\mathrm{i}k_2r+\mathrm{i}\pi/4}}{\sqrt{2\pi k_2 r}}(1+\mathrm{O}((k_2r)^{-1})) \tag{55}$$

$$D_1(\varphi) = f(-\pi+\varphi) - f(\pi+\varphi) \tag{56}$$

$$D_2(\bar{\varphi}) = g(-\pi-\bar{\varphi}) - g(\pi-\bar{\varphi}). \tag{57}$$

The functions u^c and v^c represent the cylindrical waves propagating from the edge of the wedge. The corresponding expressions are the leading contributions from the saddle points of the Sommerfeld integrals: Maliuzhinets (1958). Since we could compute the spectral functions in the form of the perturbation series, we can analytically determine the diffraction coefficients (56) and (57) in the expressions for the cylindrical waves (54) and (55). For the spectral function f we use the first two terms of the perturbation series (see (91) and (92)) and obtain

$$\begin{aligned} f(\pi+\varphi) = {} & -R(\pi-\Phi+\varphi,\gamma,\lambda)\sigma_{\varphi_0}(2\Phi-\pi-\varphi)[1+\lambda(K_1G_0)(2\Phi-\pi-\varphi)+\mathrm{O}(\lambda^2)] \\ & +\lambda T(\pi-\Phi+\varphi,\gamma,\lambda)[G_0(b(\pi-\Phi+\varphi)-\bar{\Phi})+\mathrm{O}(\lambda)] \qquad (58) \\ & \pi+\varphi\in\Pi_z(\pi-\Phi,\pi+\Phi) \\ f(-\pi+\varphi) = {} & -R(-\pi+\Phi+\varphi,\gamma,\lambda)\sigma_{\varphi_0}(-2\Phi+\pi-\varphi) \\ & \times[1+\lambda(K_1G_0)(-2\Phi+\pi-\varphi)+\mathrm{O}(\lambda^2)] \\ & +\lambda T(-\pi+\Phi+\varphi,\gamma,\lambda)[G_0(b(-\pi+\Phi+\varphi)+\bar{\Phi})+\mathrm{O}(\lambda)] \\ & -\pi+\varphi\in\Pi_z(-\pi-\Phi,-\pi+\Phi). \qquad (59) \end{aligned}$$

Note that in expressions (59) the integrals $(K_1G_0)(2\Phi-\pi-\varphi)$ and $G_0(b(\pi-\Phi+\varphi)-\bar{\Phi}) = (K_2F_0)\ (b(\pi-\Phi+\varphi)-\bar{\Phi})$, respectively, are represented by (42) and (43) if the arguments belong to the corresponding basic strips; otherwise, one should use formulae for analytic continuation. The analogous remark is valid for expression (59). However, if we consider the leading approximation only, we obtain a very simple formula for the diffraction coefficient:

$$D_1(\varphi) \sim R(\pi-\Phi+\varphi,\gamma,\lambda)\sigma_{\varphi_0}(2\Phi-\pi-\varphi) - R(-\pi+\Phi+\varphi,\gamma,\lambda)\sigma_{\varphi_0}(-2\Phi+\pi-\varphi). \qquad (60)$$

In the leading approximation the diffraction coefficient $D_1(\varphi)$ for the external field is expressed in the elementary functions and, after rearrangement, coincides with that obtained in the paper by Kim *et al* (1991). Approximate diffraction coefficient (60) has no singularity in the transition region for the doubly transmitted wave, which was expected because the doubly transmitted wave has $\mathrm{O}(\lambda)$. Expression (60) can be used for a very highly contrast wedge.

In the leading approximation the diffraction coefficient $D_2(\bar{\varphi})$ in (57) for the internal field is given by (57), where

$$\begin{aligned} g(\zeta) = {} & R(\zeta-\bar{\Phi},\gamma^{-1},\lambda^{-1})R(3\bar{\Phi}-\zeta,\gamma^{-1},\lambda^{-1})[G_0(\zeta-4\bar{\Phi})+\mathrm{O}(\lambda)] \\ & +(-\lambda^{-1})R(\zeta-\bar{\Phi},\gamma^{-1},\lambda^{-1})T(3\bar{\Phi}-\zeta,\gamma^{-1},\lambda^{-1}) \\ & \times\frac{(-\mu)\sin\mu a(3\bar{\Phi}-\zeta)}{\cos\mu a(3\bar{\Phi}-\zeta)-\sin\mu\varphi_0}[F_0+\mathrm{O}(\lambda)] \\ & +\lambda^{-1}T(\zeta-\bar{\Phi},\gamma^{-1},\lambda^{-1})\frac{\mu\sin\mu a(\zeta-\bar{\Phi})}{-\cos\mu a(\zeta-\bar{\Phi})-\sin\mu\varphi_0}[F_0+\mathrm{O}(\lambda)] \qquad (61) \end{aligned}$$

with $\zeta=\pi-\bar{\varphi}\in\Pi_\zeta(\pi-\bar{\Phi},\pi+\bar{\Phi})$, and

$$\begin{aligned} g(\zeta) = {} & R(\zeta+\bar{\Phi},\gamma^{-1},\lambda^{-1})R(-3\bar{\Phi}-\zeta,\gamma^{-1},\lambda^{-1})[G_0(\zeta+4\bar{\Phi})+\mathrm{O}(\lambda)] \\ & +(-\lambda^{-1})R(\zeta+\bar{\Phi},\gamma^{-1},\lambda^{-1})T(-3\bar{\Phi}-\zeta,\gamma^{-1},\lambda^{-1}) \\ & \times\frac{\mu\sin\mu a(-3\bar{\Phi}-\zeta)}{-\cos\mu a(-3\bar{\Phi}-\zeta)-\sin\mu\varphi_0}[F_0+\mathrm{O}(\lambda)] \\ & +\lambda^{-1}T(\zeta+\bar{\Phi},\gamma^{-1},\lambda^{-1})\frac{(-\mu)\sin\mu a(\zeta+\bar{\Phi})}{\cos\mu a(\zeta+\bar{\Phi})-\sin\mu\varphi_0}[F_0+\mathrm{O}(\lambda)] \qquad (62) \end{aligned}$$

with $\zeta=-\pi-\bar{\varphi}\in\Pi_\zeta(-\pi-\bar{\Phi},-\pi+\bar{\Phi})$ $F_0=1$. Expressions (61) and (62) depend on elementary functions as well as on s-integrals defined in the basic strip.

4.4. Branch cuts and the lateral waves

We could consider contributions from the saddle points and from poles which can be captured when deforming the Sommerfeld contours $C^{\pm}$ of integration into the steepest descent paths $C^{+\prime} \cup C^{-\prime}$. Note that the poles of the reflection and transmission coefficients in (91)–(96) are located outside the area covered in the process of the mentioned deformation of the contours. However, there are several branch cuts (figure 2) which contribute to the asymptotics. The function $f(z)$ has the branch cuts with the complex endpoints. For large $k_1 r$ due to the exponential factor in the integrand of (11) the corresponding contribution is expected to be exponentially small. The real branch cuts of $g(\zeta)$ give rise to the lateral waves inside the wedge. One can expect that these waves have the lower order in powers of $(k_2 r)^{-1}$ in comparison with the cylindrical waves and can be neglected in the leading approximation of the far field. The procedure of studying the contributions from the branch cuts in the far-field asymptotics was discussed by Petrashen' and Budaev (1986).

4.5. A uniform formula for the far field in the exterior of a very highly contrast wedge

We assume that a tranparent wedge is very highly contrast, which means that we can use formula (60) for the diffraction coefficient and neglect the doubly transmitted waves. A uniform-with-respect-to-φ formula for the far field can be deduced from the nonuniform expressions with the aid of the traditional procedure developed by Borovikov and Kinber (1994) and applied to an imperfectly conducting wedge (Ljalinov 1996). We assume that only one side of the wedge is illuminated by the incident wave ($\pi - \Phi < \varphi_0 \leqslant \Phi$). In this way we obtain

$$
\begin{aligned}
u(k_1 r, \varphi) = {} & \exp(-\mathrm{i}k_1 r \cos[\varphi - \varphi_0]) F\left(\sqrt{2k_1 r}\cos\left(\frac{\varphi - \varphi_0}{2}\right)\right) \\
& + R(\Phi - \varphi_0, \gamma, \lambda)\exp(-\mathrm{i}k_1 r\cos[2\Phi - \varphi - \varphi_0]) \\
& \times F\left(\sqrt{2k_1 r}\cos\left(\frac{2\Phi - \varphi - \varphi_0}{2}\right)\right) \\
& + \left\{\frac{1}{2\cos\left(\frac{\varphi - \varphi_0}{2}\right)} + \frac{R(\Phi - \varphi_0, \gamma, \lambda)}{2\cos\left(\frac{2\Phi - \varphi - \varphi_0}{2}\right)}\right. \\
& \left. + \frac{\mu\cos\mu(2\Phi - \pi - \varphi)R(\pi + \varphi - \Phi)}{\sin\mu(2\Phi - \pi - \varphi) - \sin\mu\varphi_0}\right\} \\
& \times \frac{\mathrm{e}^{\mathrm{i}k_1 r + \mathrm{i}\pi/4}}{\sqrt{2\pi k_1 r}} + f(-\pi + \varphi)\frac{\mathrm{e}^{\mathrm{i}k_1 r + \mathrm{i}\pi/4}}{\sqrt{2\pi k_1 r}} + \mathrm{O}\left(\frac{1}{(k_1 r)^{3/2}}\right) + \mathrm{O}(\lambda) \\
& \pi - \Phi < \varphi_0 \leqslant \Phi
\end{aligned}
$$

where $f(-\pi + \varphi) = \sigma_{\varphi_0}(-\pi + \varphi) + \mathrm{O}(\lambda)$ and $F(x)$ is the Fresnel integral (Ljalinov 1996)

$$
\begin{aligned}
F(x) = {} & \frac{1}{\sqrt{\mathrm{i}\pi}}\int_{-\infty}^{x} \mathrm{e}^{\mathrm{i}t^2}\,\mathrm{d}t \asymp H(x) \\
& -\mathrm{sign}(x)\frac{\mathrm{i}\mathrm{e}^{\mathrm{i}x^2}}{2\pi}\sum_{m=0}^{\infty}\frac{\Gamma(m + 1/2)}{(\mathrm{i}x^2)^{m+1/2}} \qquad |x| \gg 1.
\end{aligned}
$$

It is worth mentioning that the first two terms depending on the Fresnel integrals describe the wave field in the transition regions of the light–shadow boundaries for the incident wave ($\varphi - \varphi_0 = \pi$) and for the reflected wave from the left face ($\pi + \varphi = 2\Phi - \varphi_0$). These integrals degenerate in accordance with their asymptotics if the observation point is outside close vicinities of the light–shadow boundaries and we obtain nonuniform expressions for incident, reflected and scattered waves. The singularities of the other terms on the corresponding light–shadow boundaries cancel each other so that the formula is really uniform with respect to the observation angle φ. It is easily verified that the expression for the far field satisfies the explicit boundary conditions on the illuminated wedge's face $\varphi = \Phi$ and the Neumann boundary condition on the shadowed surface $\varphi = -\Phi$. We hope that construction of a uniform asymptotic expression involving higher-order terms with respect to λ (for example, the doubly transmitted wave) is also possible on the basis of the traditional procedure (Borovikov and Kinber 1994).

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Appendix A. On some extension of the theory of S-integrals

Let us consider difference equations for one unknown function $S(\alpha)$

$$S(\alpha - (-1)^j\Phi) - S(-\alpha - (-1)^j\Phi) = H_j(\alpha) \qquad \alpha \in C^* \quad j = 1, 2 \qquad (63)$$

where $H_j(\alpha)$ are given functions on C^* which are regular in a vicinity of the imaginary axis excluding points of the branch cut (figure 3(a)). We also study the similar equations

$$s(\beta - (-1)^j\bar{\Phi}) - s(-\beta - (-1)^j\bar{\Phi}) = h_j(\beta) \qquad \beta \in C_* \quad j = 1, 2 \qquad (64)$$

where $h_j(\beta)$ are defined in C_*. Solutions of equations (63) and (64) are constructed similarly and for compactness we consider system (63).

Lemma A.1. *Let $H_1(\alpha)$, $\alpha \in C^*$ be an odd function which decreases exponentially as $\alpha \to \mathrm{i}\infty$ and is regular at points of the countour L_α (figure 3(a)). Then a particular solution of the inhomogeneous system (63) (with $H_2 = 0$) which is regular in the strip $\Pi_\alpha(-3\Phi, \Phi)$ is determined by S-integral*

$$S(\alpha) = \frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{H_1(\tau)\sin\mu\tau}{\cos\mu\tau - \sin\mu\alpha}\,\mathrm{d}\tau \qquad \mu = \pi/(2\Phi) \qquad (65)$$

for $\alpha \in \Pi_\alpha(-3\Phi, \Phi)$ or by its analytic continuation

$$S(\alpha) = \frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{H_1(\tau)\sin\mu\tau - H_1(\alpha - \Phi)\sin\mu(\alpha - \Phi)}{\cos\mu\tau - \cos\mu(\alpha - \Phi)}\,\mathrm{d}\tau + (\alpha + \Phi)/(4\Phi)H_1(\alpha - \Phi) \qquad (66)$$

for $\alpha \in \Pi_\alpha(-3\Phi, 5\Phi)$. The discontinuous contour L_α goes along the imaginary axis from $-\mathrm{i}\infty$ to $+0$ and, then, from -0 to $+\mathrm{i}\infty$ (figure 3(a)).

The proof of the lemma is analogous to that obtained for a meromorphic function H_1: see Tuzhilin (1973). Formula (66) for the analytic continuation is deduced by means of the known identity (Tuzhilin 1973)

$$\frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{1}{\cos\mu\tau - \sin\mu\alpha}\,\mathrm{d}\tau = (n + \tfrac{3}{4} - \alpha/(4\Phi))/\cos\mu\alpha$$
$$\alpha \in \Pi_\alpha((4n+1)\Phi, (4n+5)\Phi)$$

where n is integer. It is obvious that in the corresponding strip $S(\alpha)$ in (66) has singularities originated from $H_1(\alpha - \Phi)$ only: in the integrand the zeros of the denominator are compensated by the zeros of the numerator for $\alpha = \tau + \Phi$. In particular, $S(\alpha)$ has the branch cut on the boundary of the basic strip passing from $\Phi + 0$ to $\Phi + \gamma^* + 0$. $S(\alpha)$ is regular inside the basic strip $\Pi_\alpha(-\Phi, \Phi)$. By the direct substitution of (66) into system (63) we verify that integral (66) is a particular solution: Tuzhilin (1973).

Lemma A.2. *Let $S_1(\alpha)$ be a particular solution of the system*

$$\begin{aligned} S(\alpha + \Phi) - S(-\alpha - \Phi) &= H(\alpha) \\ S(\alpha - \Phi) - S(-\alpha - \Phi) &= 0 \end{aligned} \tag{67}$$

with $H(\alpha) = H_1(\alpha)$ and $S_2(\alpha)$ be a particular solution of (67) with $H(\alpha) = H_2(\alpha)$. Then a particular solution of system (63) is given by the formula

$$S(\alpha) = S_1(\alpha) - S(\alpha - 2\Phi).$$

This lemma is due to Tuzhilin (1973), its proof is elementary and is omitted herein. For the exponentially decreasing $H_j(\alpha)$, $j = 1, 2$, we conclude that a particular solution of equations (63) takes on the form

$$\begin{aligned} S(\alpha) = \frac{\mathrm{i}}{8\Phi} \int_{L_\alpha} & \frac{H_1(\tau)\sin\mu\tau - H_1(\alpha - \Phi)\sin\mu(\alpha - \Phi)}{\cos\mu\tau - \cos\mu(\alpha - \Phi)}\,\mathrm{d}\tau \\ & - \frac{\mathrm{i}}{8\Phi} \int_{L_\alpha} \frac{H_2(\tau)\sin\mu\tau - H_2(\alpha + \Phi)\sin\mu(\alpha + \Phi)}{\cos\mu\tau - \cos\mu(\alpha + \Phi)}\,\mathrm{d}\tau + \frac{\alpha + \Phi}{4\Phi} H_1(\alpha - \Phi) \\ & - \frac{\alpha - \Phi}{4\Phi} H_2(\alpha + \Phi) \qquad \alpha \in \Pi_\alpha(-3\Phi, 3\Phi). \end{aligned} \tag{68}$$

In the same manner a particular solution of system (64) with the exponentially decreasing $h_j(\beta)$, $j = 1, 2$ as $\beta \to \mathrm{i}\infty$ can be written as

$$\begin{aligned} s(\beta) = \frac{\mathrm{i}}{8\bar\Phi} \int_{L_\beta} & \frac{h_1(\tau)\sin\bar\mu\tau - h_1(\beta - \bar\Phi)\sin\bar\mu(\beta - \bar\Phi)}{\cos\bar\mu\tau - \cos\bar\mu(\beta - \bar\Phi)}\,\mathrm{d}\tau \\ & - \frac{\mathrm{i}}{8\bar\Phi} \int_{L_\beta} \frac{h_2(\tau)\sin\bar\mu\tau - h_2(\beta + \bar\Phi)\sin\bar\mu(\beta + \bar\Phi)}{\cos\bar\mu\tau - \cos\bar\mu(\beta + \bar\Phi)}\,\mathrm{d}\tau + \frac{\beta + \bar\Phi}{4\bar\Phi} h_1(\beta - \bar\Phi) \\ & - \frac{\beta - \bar\Phi}{4\bar\Phi} h_2(\beta + \bar\Phi) \qquad \beta \in \Pi_\beta(-3\bar\Phi, 3\bar\Phi) \qquad \bar\mu = \frac{\pi}{2\bar\Phi} \end{aligned} \tag{69}$$

where the contour L_β is shown in figure 3(*b*). It is useful to notice that in the corresponding strips $\Pi_\alpha(-\Phi - \varepsilon, \Phi + \varepsilon)$ and $\Pi_\beta(-\bar\Phi - \varepsilon, \bar\Phi + \varepsilon)$ with sufficiently small $\varepsilon > 0$ the only singularities of $S(\alpha)$ and $s(\beta)$ are the branch cuts shown in figure 4 provided that H_j and h_j have no other singularities but the corresponding branch cuts of C^* and C_* respectively. The solution $S(\alpha)(s(\beta))$ is regular in $\Pi_\alpha(-\Phi, \Phi)(\Pi_\beta(-\bar\Phi, \bar\Phi))$.

By use of functional equations (63) we can easily obtain

$$S(\alpha + 4\Phi) = S(\alpha) + H_1(\alpha + 3\Phi) - H_2(\alpha + \Phi)$$

where $\alpha \in \Pi_\alpha(-3\Phi, 3\Phi)$. In the same manner we have

$$\begin{aligned} S(\alpha + 4n\Phi) &= S(\alpha) + \sum_{m=1}^{n} H_1(\alpha + (4m - 1)\Phi) - \sum_{m=1}^{n} H_2(\alpha + (4m - 3)\Phi) \\ S(\alpha - 4n\Phi) &= S(\alpha) - \sum_{m=1}^{n} H_1(\alpha - (4m - 3)\Phi) - \sum_{m=1}^{n} H_2(\alpha - (4m - 1)\Phi) \end{aligned} \tag{70}$$

where $n \geqslant 1$ is integer, $\alpha \in \Pi_\alpha(-3\Phi, 3\Phi)$. Formulae (70) enable us to continue $S(\alpha)$ from the strip $\alpha \in \Pi_\alpha(-3\Phi, 3\Phi)$ onto the whole α-plane. From (70) and (68) for the strips

$\Pi_\alpha((4n-3)\Phi, (4n+3)\Phi)$, $n \geqslant 0$ we deduce an explicit analytic representation

$$S(\alpha) = \frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{H_1(\tau)\sin\mu\tau - H_1(\alpha-(4n+1)\Phi)\sin\mu(\alpha-(4n+1)\Phi)}{\cos\mu\tau - \cos\mu(\alpha-(4n+1)\Phi)}\,\mathrm{d}\tau$$
$$-\frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{H_2(\tau)\sin\mu\tau - H_2(\alpha-(4n-1)\Phi)\sin\mu(\alpha-(4n-1)\Phi)}{\cos\mu\tau - \cos\mu(\alpha-(4n-1)\Phi)}\,\mathrm{d}\tau$$
$$+\frac{\alpha-(4n-1)\Phi}{4\Phi}H_1(\alpha-(4n+1)\Phi) - \frac{\alpha-(4n+1)\Phi}{4\Phi}H_2(\alpha-(4n-1)\Phi)$$
$$+\sum_{m=1}^{n}[H_1(\alpha-(4m-3)\Phi) - H_2(\alpha-(4m-1)\Phi)] \tag{71}$$

and for the strip $\alpha \in \Pi_\alpha(-(4n+3)\Phi, -(4n-3)\Phi)$

$$S(\alpha) = \frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{H_1(\tau)\sin\mu\tau - H_1(\alpha+(4n-1)\Phi)\sin\mu(\alpha+(4n-1)\Phi)}{\cos\mu\tau - \cos\mu(\alpha+(4n-1)\Phi)}\,\mathrm{d}\tau$$
$$-\frac{\mathrm{i}}{8\Phi}\int_{L_\alpha} \frac{H_2(\tau)\sin\mu\tau - H_2(\alpha+(4n+1)\Phi)\sin\mu(\alpha+(4n+1)\Phi)}{\cos\mu\tau - \cos\mu(\alpha+(4n+1)\Phi)}\,\mathrm{d}\tau$$
$$+\frac{\alpha+(4n+1)\Phi}{4\Phi}H_1(\alpha+(4n-1)\Phi) - \frac{\alpha+(4n-1)\Phi}{4\Phi}H_2(\alpha+(4n+1)\Phi)$$
$$-\sum_{m=1}^{n}[H_1(\alpha+(4m-1)\Phi) - H_2(\alpha+(4m-3)\Phi)]. \tag{72}$$

The solution $s(\beta)$ can be represented in a form similar to (71), (72).

If the right-hand side of (63) grows no faster than an exponent $\exp(\nu|\alpha|)$ as $\alpha \to \pm\mathrm{i}\infty$, we can substitute $S(\alpha) = \sin^m \mu\alpha S_0(\alpha)$ with a sufficiently large integer m, $\mu m > \nu$ so that the system

$$S_0(\alpha - (-1)^j\Phi) - S_0(-\alpha - (-1)^j\Phi) = (-1)^{m(j+1)}H_j(\alpha)/\cos^m\mu\alpha \qquad j = 1, 2 \tag{73}$$

has an exponentially decreasing right-hand side and the above-discussed theory can be applied. We use this theory in order to reduce the system of functional equations (23) to a system of linear equaitons in a Banach space.

Appendix B. Proof of boundedness of the operators K_1 and K_2

First, we prove the boundedness of the operator K_2. Consider the estimate

$$I = \sup_{\beta\in\Pi_2}|\nu_\delta(\beta)[(K_2\xi)(\beta) - \kappa(\beta)(K_2\xi)(\mathrm{i}\infty)]|$$
$$= \sup_{\beta\in\partial\Pi_2}|\nu_\delta(\beta)[(K_2\xi)(\beta) - \kappa(\beta)(K_2\xi)(\mathrm{i}\infty)]|$$
$$\leqslant C_0(\sup_{x\in R\backslash 0}|\nu_\delta(\mathrm{i}x + \bar\Phi + \varepsilon)[(K_2\xi)(\mathrm{i}x + \bar\Phi + \varepsilon) - \kappa(\mathrm{i}x + \bar\Phi + \varepsilon)(K_2\xi)(\mathrm{i}\infty)]|$$
$$+\sup_{x\in R\backslash 0}|\nu_\delta(\mathrm{i}x - \bar\Phi - \varepsilon)[(K_2\xi)(\mathrm{i}x - \bar\Phi - \varepsilon)$$
$$-\kappa(\mathrm{i}x - \bar\Phi - \varepsilon)(K_2\xi)(\mathrm{i}\infty)]|) = C_0(I_1 + I_2). \tag{74}$$

In estimate (74), first, we exploited the well known result that a regular function attains its maximal value on the boundary of a domain and, then, the supremum I was estimated by suprema over the right vertical boundary of Π_2 and over the left one. These suprema are denoted as I_1 and I_2 respectively. We demonstrate an estimate for I_1, then I_2 is treated in the similar manner. By use of the formulae for the analytic continuation of s-integrals we can identically transform I_1 in (74) to the form

$$I_1 = \sup_{x\in R\backslash 0}\Big|\nu_\delta(\mathrm{i}x + \bar\Phi + \varepsilon)$$

$$\times\left\{\frac{\mathrm{i}}{8\bar\Phi}\int_{L_\beta}\mathrm{d}\tau\left(\frac{D_2(\tau,\xi)\cos\bar\mu\tau}{\cos\bar\mu\tau+\cos\bar\mu(\mathrm{i}x+\varepsilon)}-\frac{D_1(\tau,\xi)\cos\bar\mu\tau}{\cos\bar\mu\tau-\cos\bar\mu(\mathrm{i}x+\varepsilon)}\right)\right.$$
$$+D_1(\mathrm{i}x+\varepsilon,\xi)\cot\bar\mu(\mathrm{i}x+\varepsilon)\frac{\mathrm{i}x+\varepsilon-2\bar\Phi}{4\bar\Phi}$$
$$\left.\left.-D_2(\mathrm{i}x+\varepsilon,\xi)\cot\bar\mu(\mathrm{i}x+\varepsilon)\frac{\mathrm{i}x+\varepsilon+2\bar\Phi}{4\bar\Phi}-\kappa(\mathrm{i}x+\bar\Phi+\varepsilon)(-\mathrm{i}\mu\xi(\mathrm{i}\infty))\right\}\right|$$
$$=\sup_{x\in R\setminus 0}\left|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)\left\{\frac{\mathrm{i}}{8\bar\Phi}\int_{L_\beta}\mathrm{d}\tau\right.\right.$$
$$\times\Big[\mu\psi_+(\tau)[\xi(a(\tau)-\Phi)+\xi(-a(\tau)-\Phi)]+2\mu\xi(\mathrm{i}\infty))$$
$$\times\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau+\cos\bar\mu(\mathrm{i}x+\bar\Phi)}-(\mu\psi_-(\tau)[\xi(a(\tau)+\Phi)+\xi(-a(\tau)-\Phi)]$$
$$+2\mu\xi(\mathrm{i}\infty))\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau-\cos\bar\mu(\mathrm{i}x+\bar\Phi)}\Big]$$
$$+\frac{\mathrm{i}}{8\bar\Phi}\int_{L_\beta}\left(\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau-\cos\bar\mu(\mathrm{i}x+\bar\Phi)}-\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau+\cos\bar\mu(\mathrm{i}x+\bar\Phi)}\right)\mathrm{d}\tau$$
$$\times 2\mu\xi(\mathrm{i}\infty)+(\mu\psi_-(\mathrm{i}x+\varepsilon)[\xi(a(\mathrm{i}x+\varepsilon)+\Phi)$$
$$+\xi(-a(\mathrm{i}x+\varepsilon)+\Phi)]-\mu\psi_+(\mathrm{i}x+\varepsilon)[\xi(a(\mathrm{i}x+\varepsilon)-\Phi)$$
$$+\xi(-a(\mathrm{i}x-\varepsilon)-\Phi)])\cot\bar\mu(\mathrm{i}x+\varepsilon)\frac{\mathrm{i}x+\varepsilon}{4\bar\Phi}$$
$$+(-\mu/2)\psi_-(\mathrm{i}x+\varepsilon)[\xi(a(\mathrm{i}x+\varepsilon)+\Phi)+\xi(-a(\mathrm{i}x+\varepsilon)+\Phi)]\cot\bar\mu(\mathrm{i}x+\varepsilon)$$
$$+(-\mu/2)\psi_+(\mathrm{i}x+\varepsilon)[\xi(a(\mathrm{i}x+\varepsilon)-\Phi)+\xi(-a(\mathrm{i}x+\varepsilon)-\Phi)]\cot\bar\mu(\mathrm{i}x+\varepsilon)$$
$$\left.\left.-\kappa(\mathrm{i}x+\bar\Phi+\varepsilon)(-\mathrm{i}\mu\xi(\mathrm{i}\infty))\right\}\right|\qquad \xi\in A_1 \tag{75}$$

where we used $D(\mathrm{i}\infty,\xi)=-\mathrm{i}\mu\xi(\mathrm{i}\infty)$,

$$\psi_\pm(\tau)=\frac{\sin\tau}{\sqrt{\gamma^2-\cos^2\tau}}\frac{\sin\mu a(\tau)\tan\bar\mu\tau}{\cos\mu a(\tau)\pm\sin\mu\varphi_0}$$

with $\psi_\pm(\tau)\to -1$ and

$$|\psi_\pm(\tau)+1|\leqslant \mathrm{const}\,\mathrm{e}^{-\mu|\mathrm{Im}\,\tau|} \tag{76}$$

as $\tau\to\pm\mathrm{i}\infty$; $\mu<1$, $\mu<\bar\mu$. The last integral in the right-hand side of equalities (75) can be explicitely calculated:

$$\frac{\mathrm{i}}{8\bar\Phi}\int_{L_\beta}\left(\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau-\cos\bar\mu(\mathrm{i}x+\varepsilon)}-\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau+\cos\bar\mu(\mathrm{i}x+\varepsilon)}\right)\mathrm{d}\tau=-\tfrac{1}{2}\cot\bar\mu(\mathrm{i}x+\varepsilon).$$

After simple rearrangements we come to the form of I_1 which is subjected to further estimation

$$I_1\leqslant I_1^-+I_1^++I_1^0 \tag{77}$$

with

$$I_1^\pm=\sup_{x\in R}\left|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)\frac{\pm\mathrm{i}\mu}{8\bar\Phi}\int_{L_\beta}\mathrm{d}\tau\,(\psi_\pm(\tau)[\xi(a(\tau)\mp\Phi)-\xi(\mathrm{i}\infty)\right.$$
$$\left.+\xi(-a(\tau)\mp\Phi)-\xi(\mathrm{i}\infty)]+2(\psi_\pm(\tau)+1))\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau\pm\cos\bar\mu(\mathrm{i}x+\varepsilon)}\right| \tag{78}$$

and

$$I_1^0 = \sup_{x\in R\setminus 0}|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)\{(\psi_-(\mathrm{i}x+\varepsilon)[\xi(a(x+\varepsilon)+\Phi)+\xi(-a(\mathrm{i}x+\varepsilon)+\Phi)]$$
$$-\psi_+(\mathrm{i}x+\varepsilon)[\xi(a(\mathrm{i}x+\varepsilon)-\Phi)+\xi(-a(\mathrm{i}x+\varepsilon)-\Phi)])$$
$$\times\frac{\mu(\mathrm{i}x+\varepsilon)}{4\bar\Phi}\cot\bar\mu(\mathrm{i}x+\varepsilon)-\mu/2\cot\bar\mu(\mathrm{i}x+\varepsilon)(\psi_-(\mathrm{i}x+\varepsilon)[\xi(a(\mathrm{i}x+\varepsilon)+\Phi)$$
$$+\xi(a(\mathrm{i}x+\varepsilon)+\Phi)]+2\xi(\mathrm{i}\infty))-\mu/2\cot\bar\mu(\mathrm{i}x+\varepsilon)(\psi_+(\mathrm{i}x+\varepsilon)$$
$$\times[\xi(a(\mathrm{i}x+\varepsilon)-\Phi)+\xi(a(\mathrm{i}x+\varepsilon)-\Phi)]$$
$$+2\xi(\mathrm{i}\infty))+\mu\xi(\mathrm{i}\infty)\cot\bar\mu(\mathrm{i}x+\varepsilon)(1+\mathrm{i}\tan\bar\mu(\mathrm{i}x+\varepsilon)\kappa(\mathrm{i}x+\bar\Phi+\varepsilon))\}|.$$

Taking into account estimate (76) and the asymptotic behaviour of $\kappa(\beta)$, by use of the definition of the norm $\|\dot{;}A_1\|$ and by inequality $|\xi(\mathrm{i}\infty)|\leqslant\|\xi;A_1\|$ we come to the estimate for I_1^0:

$$I_1^0\leqslant C_1^0\|\xi;A_1\|. \tag{79}$$

Now we turn to $I_1^\pm$. We have

$$I_1^\pm\leqslant J_1^\pm\|\xi;A_1\|+J_2^\pm\|\xi;A_1\| \tag{80}$$

with

$$J_1^\pm=\sup_{x\in R}\left\{\frac{\mu}{8\bar\Phi}\int_{L_\beta}|\,\mathrm{d}\tau\,||\psi_\pm(\tau)|\frac{|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)|}{|\nu_\delta(\tau)|}\left|\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau\pm\cos\bar\mu(\mathrm{i}x+\varepsilon)}\right|\right\} \tag{81}$$

$$J_2^\pm=\sup_{x\in R}\left\{\frac{\mu}{4\bar\Phi}\int_{L_\beta}|\,\mathrm{d}\tau\,||\nu_\delta(\tau)(\psi_\pm(\tau)+1)|\frac{|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)|}{|\nu_\delta(\tau)|}\left|\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau\pm\cos\bar\mu(\mathrm{i}x+\varepsilon)}\right|\right\}. \tag{82}$$

One can prove the following lemma.

Lemma B.1. *The suprema $J_{1,2}^\pm$ in (81) and (82) are bounded if $\delta<\mu$.*

Proof of this lemma follows from boundedness of the supremum

$$J=\sup_{x\in R}\left\{\int_{L_\beta}|\,\mathrm{d}\tau\,|\frac{|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)|}{|\nu_\delta(\tau)|}\left|\frac{\cos\bar\mu\tau}{\cos\bar\mu\tau\pm\cos\bar\mu(\mathrm{i}x+\varepsilon)}\right|\right\} \tag{83}$$

since in accordance with (76) the inequalities

$$|\psi_\pm(\tau)|<\mathrm{const}$$
$$|\nu_\delta(\tau)(\psi_\pm(\tau)+1)|<\mathrm{const}\qquad(\delta<\mu)$$

on L_β is valid. In order to prove the boundedness of J we introduce the new variable of integration $w=\tau/\mathrm{i}$ in (83) and exploit the obvious estimates for ν_δ:

$$|\nu_\delta(\mathrm{i}x+\bar\Phi+\varepsilon)|\leqslant C\exp(\delta|x|)\leqslant 2c\cosh(\delta x)\qquad x\in\mathbb{R}$$
$$d_0+d_1\exp(\delta|w|)\leqslant|\nu_\delta(\mathrm{i}w)|\leqslant d_2\exp(\delta|w|)\qquad w\in\mathbb{R}$$

where C, d_0, d_1 and d_2 are positive constants. After simple transform we come to the chain of estimates ($\bar\sigma=\delta/\bar\mu$)

$$J\leqslant 2C\sup_{x\in R}\int_0^\infty\left\{\frac{\cosh(\bar\mu w)[\exp(\bar\mu|x|)]^{\bar\sigma}/(d_0+d_1[\exp(\bar\mu w/2)]^{\bar\sigma})}{[(\cosh\bar\mu w-\cos\bar\mu\varepsilon\cosh\bar\mu x)^2+\sinh^2\bar\mu x\sin^2\bar\mu\varepsilon]^{1/2}}\right\}\mathrm{d}w$$
$$\leqslant C_1\sup_{x\in R}\int_0^\infty\left\{\frac{\cosh(\bar\mu w)[\cosh(\bar\mu|x|)]^{\bar\sigma}/(d_0+d_1[\cosh(\bar\mu w/2)]^{\bar\sigma})}{[(\cosh\bar\mu w-\cos\bar\mu\varepsilon\cosh\bar\mu x)^2+\sinh^2\bar\mu x\sin^2\bar\mu\varepsilon]^{1/2}}\right\}\mathrm{d}w$$
$$=C_2\sup_{q\geqslant 1}\int_0^\infty\frac{[q]^{\bar\sigma}/(d_0+d_1[\sqrt{1+p^2}/2)]^{\bar\sigma})}{[(\sqrt{1+p^2}-q\cos\bar\mu\varepsilon)^2+(q^2-1)\sin^2\bar\mu\varepsilon]^{1/2}}\,\mathrm{d}p. \tag{84}$$

In the last step we have used the new integration variable $p = \sinh(\bar{\mu}w)$ and notation $q = \cosh(\bar{\mu}x)$. After introducing $\tau = p/q$ the integral in the right-hand side of the chain of inequalities (84) can be split into two integrals over the intervals $[1, \infty)$ and $[0, 1]$ respectively, then we estimate the integrand in the second integral and obtain

$$J \leqslant C_3 \sup_{q\geqslant 1}\left\{ \int_1^\infty \frac{1/(d_0 + d_1[\sqrt{q^{-2}+\tau^2}/2)]^{\bar{\sigma}})}{[(\sqrt{q^{-2}+\tau^2} - \cos\bar{\mu}\varepsilon)^2 - (q^{-2}-1)\sin^2\bar{\mu}\varepsilon]^{1/2}}\,\mathrm{d}\tau\right\}$$
$$+C_4 \sup_{q\geqslant 1}\left\{\int_0^1 \frac{[q]^{\bar{\sigma}}}{(d_0 + d_1[q\tau/2)]^{\bar{\sigma}})}\,\mathrm{d}\tau\right\}. \tag{85}$$

The first summand in the right-hand side of (85) is obviously bounded, the second one is also bounded in view of the estimate

$$\int_0^1 \frac{[q]^{\bar{\sigma}}}{(d_0 + d_1[q\tau/2)]^{\bar{\sigma}})}\,\mathrm{d}\tau = q^{\bar{\sigma}-1}\int_0^q \frac{1}{(d_0 + d_1[t/2]^{\bar{\sigma}})}\,\mathrm{d}t \sim Cq^{\bar{\sigma}-1}t^{1-\bar{\sigma}}|_q \sim \mathrm{const}$$

with $\bar{\sigma} < 1$, $q \geqslant 1$. This completes the proof of the lemma.

Now the desired estimate for I_1 and I in (74) follows from (77) and (79), (80). If we take into account that

$$\|K_2\xi; A_2\| = \sup_{\beta\in\Pi_2}|(K_2\xi)(\beta)| + I \leqslant \sup_{\beta\in\Pi_2}|(K_2\xi)(\beta) - \kappa(\beta)(K_2\xi)(\mathrm{i}\infty)|$$
$$+\sup_{\beta\in\Pi_2}|(K_2\xi)(\mathrm{i}\infty)| + I \leqslant C_1 I \tag{86}$$

from the estimate for I we easily conclude that

$$\|K_2\xi; A_2\| \leqslant \mathrm{const}\|\xi; A_1\|$$

which means boundedness of the operator K_2.

In order to estimate the norm of the operator K_1 it is convenient to follow a slightly different method. We shall briefly describe it herein. As in the case of the operator K_2 (see (86), (74)) boundedness of K_1 is easily deduced from the estimate

$$N := \sup_{\alpha\in\Pi_1}|\nu_\delta(\alpha)(\tilde{K}_1\eta)(\alpha)| \leqslant \mathrm{const}\|\eta; A_2\| \tag{87}$$

where

$$(\tilde{K}_1\eta)(\alpha) = (K_1\eta)(\alpha) - (K_1\eta)(\mathrm{i}\infty)$$

with

$$(K_1\eta)(\mathrm{i}\infty) = \frac{\mathrm{i}}{8\Phi}\sum_{j=1}^{2}\int_{L_\alpha}(-1)^{j+1}/\mu[\eta(b(\tau) + (-1)^j\bar{\Phi}) + \eta(-b(\tau) + (-1)^j\bar{\Phi})]\,\mathrm{d}\tau.$$

Analogously to (74) we obtain the estimate

$$N := \sup_{\alpha\in\partial\Pi_1}|\nu_\delta(\alpha)(\tilde{K}_1\eta)(\alpha)| \leqslant \sup_{x\in R}|\nu_\delta(\mathrm{i}x + \Phi + \varepsilon)(\tilde{K}_1\eta)(\mathrm{i}x + \Phi + \varepsilon)|$$
$$+\sup_{x\in R}|\nu_\delta(\mathrm{i}x - \Phi - \varepsilon)(\tilde{K}_1\eta)(\mathrm{i}x - \Phi - \varepsilon)| = N_1 + N_2$$

where N_1 and N_2 denote the first and second summands respectively in the left-hand side of the last equality. The estimates for both of them are similar and for compactness we consider N_1 only. From the functional equations for S-integral $(\tilde{K}_1\eta)(\alpha)$ we have

$$(\tilde{K}_1\eta)(\mathrm{i}x + \Phi + \varepsilon) = (\tilde{K}_1\eta)(-\mathrm{i}x + \Phi - \varepsilon)$$
$$-\frac{\cos\mu(\mathrm{i}x + \varepsilon) - \sin\mu\varphi_0}{\mu\sin\mu(\mathrm{i}x + \varepsilon)}[\eta(b(\mathrm{i}x + \varepsilon) - \bar{\Phi}) + \eta(-b(\mathrm{i}x + \varepsilon) - \bar{\Phi})]$$

and therefore

$$N_1 \leqslant \sup_{x\in R}|\nu_\delta(\mathrm{i}x+\Phi+\varepsilon)(\tilde{K}_1\eta)(-\mathrm{i}x+\Phi-\varepsilon)| + \sup_{x\in R}\Bigg|\nu_\delta(\mathrm{i}x+\Phi+\varepsilon)$$
$$\times\frac{\cos\mu(\mathrm{i}x+\varepsilon)-\sin\mu\varphi_0}{\mu\sin\mu(\mathrm{i}x+\varepsilon)}[\eta(b(\mathrm{i}x+\varepsilon)-\bar{\Phi})+\eta(-b(\mathrm{i}x+\varepsilon)-\bar{\Phi})]\Bigg|$$
$$= N_1^1 + N_1^2 \qquad (88)$$

where N_1^1 denotes the first summand and N_1^2 the second one in the left-hand side of equality (88). We easily conclude that

$$N_1^2 \leqslant \mathrm{const}\|\eta;\, A_2\| \qquad (89)$$

where we took into account the equality

$$\eta(\beta-\bar{\Phi})+\eta(-\beta-\bar{\Phi}) = \eta(\beta-\bar{\Phi})-\kappa(\beta)\eta(\mathrm{i}\infty)+\eta(-\beta-\bar{\Phi})-\kappa(\beta)\eta(-\mathrm{i}\infty)$$

and the definition of the norm $\|.;\, A_2\|$. For N_1^1 we obtain

$$N_1^1 \leqslant \sup_{x\in R}\Bigg\{\frac{1}{8\Phi}\sum_{j=1}^{2}\int_{L_\alpha}|\mathrm{d}\tau|\left|\frac{\cos\mu\tau+(-1)^j\sin\mu\varphi_0}{\cos\mu\tau+(-1)^j\cos\mu(\mathrm{i}x+\varepsilon)}\right|\left|\frac{\nu_\delta(\mathrm{i}x+\Phi+\varepsilon)}{\nu_\delta(\tau)}\right|$$
$$\times|\nu_\delta(\tau)||\eta(b(\tau)+(-1)^j\Phi)-\kappa(\tau)\eta(\mathrm{i}\infty)$$
$$+\eta(-b(\tau)+(-1)^j\Phi)-\kappa(\tau)\eta(-\mathrm{i}\infty)|\Bigg\} \leqslant C_0 J_0\|\eta;\, A_2\| \qquad (90)$$

with

$$J_0 = \sup_{x\in R}\Bigg\{\frac{1}{8\Phi}\sum_{j=1}^{2}\int_{L_\alpha}|\mathrm{d}\tau|\left|\frac{\cos\mu\tau+(-1)^j\sin\mu\varphi_0}{\cos\mu\tau+(-1)^j\cos\mu(\mathrm{i}x+\varepsilon)}\right|\left|\frac{\nu_\delta(\mathrm{i}x+\Phi+\varepsilon)}{\nu_\delta(\tau)}\right|\Bigg\}.$$

Note that for $\delta<\mu$ boundedness of J_0 is proved in the same manner as for J in the lemma B1. From (89), (90) and (88) we come to the inequality (87) and, then, to boundedness of the operator K_1, which completes the proof of proposition 3.2.

Appendix C. Analytic continuations for the spectral functions $f(z)$ and $g(z)$

From the functional equations (20) we find

$$f(z) = -R(z-\Phi,\gamma,\lambda)\sigma_{\varphi_0}(2\Phi-z)[1+\lambda(K_1G_0)(2\Phi-z)+\mathrm{O}(\lambda^2)]$$
$$+\lambda T(z-\Phi,\gamma,\lambda)[G_0(b(z-\Phi)-\bar{\Phi})+\mathrm{O}(\lambda)] \qquad z\in\Pi_z(\Phi,3\Phi) \qquad (91)$$
$$f(z) = -R(z+\Phi,\gamma,\lambda)\sigma_{\varphi_0}(-2\Phi-z)[1+\lambda(K_1G_0)(-2\Phi-z)+\mathrm{O}(\lambda^2)]$$
$$+\lambda T(z+\Phi,\gamma,\lambda)[G_0(b(z+\Phi)+\bar{\Phi})+\mathrm{O}(\lambda)] \qquad z\in\Pi_z(-3\Phi,-\Phi). \qquad (92)$$

For $g(\zeta)$ we have

$$g(\zeta) = -R(\zeta-\bar{\Phi},\gamma^{-1},\lambda^{-1})[G_0(2\bar{\Phi}-\zeta)+\mathrm{O}(\lambda)]$$
$$+\lambda^{-1}T(\zeta-\bar{\Phi},\gamma^{-1},\lambda^{-1})\frac{\mu\sin\mu a(\zeta-\bar{\Phi})}{-\cos\mu a(\zeta-\bar{\Phi})-\sin\mu\varphi_0}[F_0+\mathrm{O}(\lambda)] \qquad (93)$$
$$\zeta\in\Pi_\zeta(\bar{\Phi},3\bar{\Phi})$$
$$g(\zeta) = -R(\zeta+\bar{\Phi},\gamma^{-1},\lambda^{-1})[G_0(-2\bar{\Phi}-\zeta)+\mathrm{O}(\lambda)]$$
$$+\lambda^{-1}T(\zeta+\bar{\Phi},\gamma^{-1},\lambda^{-1})\frac{-\mu\sin\mu a(\zeta+\bar{\Phi})}{\cos\mu a(\zeta+\bar{\Phi})-\sin\mu\varphi_0}[F_0+\mathrm{O}(\lambda)] \qquad (94)$$
$$\zeta\in\Pi_\zeta(-3\bar{\Phi},-\bar{\Phi}).$$

Since we need to compute $g(\zeta)$ in $\Pi_\zeta(-\pi-\bar\Phi, \pi+\bar\Phi)$ with $\bar\Phi = \pi/4$, two additional formulae for continuaton should be taken into account:

$$
\begin{aligned}
g(\zeta) = {} & R(\zeta - \bar\Phi, \gamma^{-1}, \lambda^{-1}) R(3\bar\Phi - \zeta, \gamma^{-1}, \lambda^{-1})[G_0(\zeta - 4\bar\Phi) + \mathrm{O}(\lambda)] \\
& +(-\lambda^{-1}) R(\zeta - \bar\Phi, \gamma^{-1}, \lambda^{-1}) T(3\bar\Phi - \zeta, \gamma^{-1}, \lambda^{-1}) \\
& \times \frac{(-\mu)\sin\mu a(3\bar\Phi - \zeta)}{\cos\mu a(3\bar\Phi - \zeta) - \sin\mu\varphi_0}[F_0 + \mathrm{O}(\lambda)] \\
& +\lambda^{-1} T(\zeta - \bar\Phi, \gamma^{-1}, \lambda^{-1}) \frac{\mu \sin\mu a(\zeta - \bar\Phi)}{-\cos\mu a(\zeta - \bar\Phi) - \sin\mu\varphi_0}[F_0 + \mathrm{O}(\lambda)] \qquad (95) \\
& \zeta \in \Pi_\zeta(3\bar\Phi, 5\bar\Phi)
\end{aligned}
$$

$$
\begin{aligned}
g(\zeta) = {} & R(\zeta + \bar\Phi, \gamma^{-1}, \lambda^{-1}) R(-3\bar\Phi - \zeta, \gamma^{-1}, \lambda^{-1})[G_0(\zeta + 4\bar\Phi) + \mathrm{O}(\lambda)] \\
& +(-\lambda^{-1}) R(\zeta + \bar\Phi, \gamma^{-1}, \lambda^{-1}) T(-3\bar\Phi - \zeta, \gamma^{-1}, \lambda^{-1}) \\
& \times \frac{\mu\sin\mu a(-3\bar\Phi - \zeta)}{-\cos\mu a(-3\bar\Phi - \zeta) - \sin\mu\varphi_0}[F_0 + \mathrm{O}(\lambda)] \\
& +\lambda^{-1} T(\zeta + \bar\Phi, \gamma^{-1}, \lambda^{-1}) \frac{(-\mu) \sin\mu a(\zeta + \bar\Phi)}{\cos\mu a(\zeta + \bar\Phi) - \sin\mu\varphi_0}[F_0 + \mathrm{O}(\lambda)] \qquad (96) \\
& \zeta \in \Pi_\zeta(-5\bar\Phi, -3\bar\Phi).
\end{aligned}
$$

Note that, since the right-hand sides in formulae (51)–(96) are known in an explicit form, their values as well as the singularities are also determined. The integrals G_0 in the right-hand sides of (93)–(96) and K_1G_0 in (91), (92) are regular functions in the corresponding basic strips, therefore, only singularities of the other factors in the right-hand sides of (51)–(96) should be taken into account. Since these factors are elementary functions, determination of their singularities which are poles and branch cuts is a simple task.

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